Evaluation of existing Compressed Sensing reconstruction algorithms on natural images

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Chapter 1

Introduction

Compressed Sensing (CS) is a framework, that deals with the reconstruction of signals with specific properties from only a small set of measurements. It builds upon the groundbreaking work from Candes et al. [3] and Donoho [5], who showed that as long as a sparse representation of the signal exists, it can be exactly reconstructed from only a small set of samples, measured in a linear fashion from the signal. The number of samples that have to be obtained for the complete representation can be much smaller than what the classical Shannon-Nyquist Theorem states [20]. CS thus allows to combine signal acquisition and dimensionality reduction at the same time. In this work, CS principles are evaluated for sensing and reconstructing natural images. At first a short overview of general CS is given in Chapter 2. Then Chapter 3 deals with published work on the topic of applying CS to natural images. Subsequently the algorithms evaluated in this work are presented in Chapter 4. The experimental results and a comparison of the different CS sensing and reconstruction algorithms are outlined in Chapter 6.
Chapter 2

Basics of Compressed Sensing

The basics of CS deal with the statistically sufficient measurement and exact reconstruction of finite, real, discrete and strictly sparse signals. As a measurement system only linear systems are considered [3], [5]. More formally this is specified as follows.

Given a vector $x$ element of $\Sigma_k$, where $\Sigma_k$ is the set of all $k$-sparse vectors for a given dimensionality $N$, i.e., $\Sigma_k = \{ v \in \mathbb{R}^N | \|v\|_0 \leq k \}$, reconstruct $x$ given only a measurement vector

$$y = \Phi x$$

(2.1)

and the measurement matrix $\Phi$. $\Phi$ is element of $\mathbb{R}^{M \times N}$, where $M \ll N$, and thus $y$ is element of $\mathbb{R}^M$. This results in an under-determined linear system of equations with sparsity as prior information about the vector to be recovered. Figure 2.1 shows an example of a measurement process. The measurement vector $y \in \mathbb{R}^M$ is the result of a matrix vector multiplication of the k-sparse input vector $x \in \Sigma_k \subseteq \mathbb{R}^N$, that has to be acquired, with the measurement matrix $\Phi \in \mathbb{R}^{M \times N}$. As $M \ll N$ the measurement
Figure 2.1: Illustration of the measurement process.

vector consists of fewer samples than the k-sparse vector.

CS states, that an exact recovery of \( \mathbf{x} \) is possible, if \( \Phi \) fulfills certain properties and a recovery process is given by

\[
x = \arg\min_{\hat{x} \in \mathbb{R}^N} \| \hat{x} \|_0, \quad s.t. \quad \mathbf{y} = \Phi \hat{x}.
\] (2.2)

However, this \( \ell_0 \) optimization has a high computational complexity and thus a large number of alternative optimizations have been proposed in the literature. Probably the most prominent of these is the Basis Pursuit (BP), which replaces the \( \ell_0 \) optimization with its closest convex norm, the \( \ell_1 \) optimization:

\[
x = \arg\min_{\hat{x} \in \mathbb{R}^N} \| \hat{x} \|_1, \quad s.t. \quad \mathbf{y} = \Phi \hat{x}.
\] (2.3)

This is a convex optimization problem and can then be recast into a linear program (LP) and solved efficiently, using existing LP algorithms. The \( \ell_1 \) optimization also guarantees perfect reconstruction for \( k \)-sparse vectors, if certain conditions for the measurement matrix are met ([3], [5]). These conditions guarantee that for each pair of different signals \( \mathbf{x}, \mathbf{x}' \in \Sigma_k \) with \( \mathbf{x} \neq \mathbf{x}' \), the measurement results in two different
measurement vectors $\Phi x \neq \Phi x'$. One property of a matrix $\Phi$ that is important in this context is the spark of a matrix. If the spark of a matrix, which is the smallest number of columns of the matrix that are linearly dependent, is greater than $2k$, then a unique measurement vector for each sparse input vector is guaranteed [6]. As the computation of the spark of a general matrix is computationally expensive, it is preferable to use different properties than the spark to provide recovery guarantees. One of these is the coherence of a matrix. The coherence $\mu(\Phi)$ is the largest absolute inner product between any two columns of $\Phi$. Now if

$$k < \frac{1}{2} \left( 1 + \frac{1}{\mu(\Phi)} \right),$$

(2.4)

the uniqueness of the measurement vector for each sparse input vector is guaranteed [6]. Another property, providing recovery guarantees is the $(k, \delta)$-restricted isometry property (RIP). It is fulfilled for the matrix $\Phi$ if

$$(1 - \delta) \|x\|_2^2 \leq \|\Phi x\|_2^2 \leq (1 + \delta) \|x\|_2^2.$$  

(2.5)

The RIP is closely linked to the coherence and spark of a matrix and additionally provides guarantees for the stability of the recovery process in the presence of noise, i.e., the distance between the measurement vector of two different sparse signal should be proportional to the distance between the sparse signals itself. Thus small noise leads to a recovered signal that has only a small distance to the original signal. This stability is guaranteed for different noise models like additive noise in the sensing stage ($y = \Phi x + n$), multiplicative noise due to a mismatch between the sensing matrix and reconstruction matrix ($\Phi_{rec} = \Phi_{sens} + \Delta$) or noise prior to measurement ($y = \Phi(x + n)$) [7]. For boundaries of the reconstruction error under different noise models see e.g. [7]. Matrices, that fulfill the constraining properties for the measurement matrix, even when $M \ll N$, are, among others, shown to be random Gaussian Matrices or random Partial Fourier Matrices. These random matrices fulfill the properties with
As signals sometimes are not directly sparse in the given basis, but have a sparse representation in a different basis, i.e., there exists an invertible transform matrix $\Psi$ so that $\tilde{x} = \Psi x$ and now $\tilde{x} \in \Sigma_k$ and $x$ does not have to be sparse directly. The measurement can still be applied directly to the signal $y = \Phi x$, but the reconstruction needs to be reformulated to

$$\tilde{x} = \arg\min_{\tilde{x} \in \mathbb{R}^N} \|\tilde{x}\|_1, \quad s.t. \quad y = \Phi \Psi^{-1} \tilde{x}, \quad (2.6)$$

and the final reconstruction is

$$x = \Psi^{-1} \tilde{x}. \quad (2.7)$$

Figure 2.2 shows the relation between the different signals and matrices. The recon-
struction matrix $\Phi \Psi^{-1}$ that is now used for reconstruction has to adhere to the same properties, as previously $\Phi$ had to. If $\Phi$ is a random Gaussian Matrix then the properties still hold for $\Phi \Psi^{-1}$ with a high probability ([3], [5]).

If the measurement process allows to incorporate the transform matrix, the sparsi-fying transform can also be done before applying the measurement matrix ($y = \Phi \Psi x$). The reconstruction of the transformed sparse vector is then:

$$\tilde{x} = \arg\min_{\hat{x} \in \mathbb{R}^N} \|\hat{x}\|_1, \quad s.t. \quad y = \Phi \hat{x}. \quad (2.8)$$

It has to be noted, that many signals in the real-world are not strictly sparse in any basis, but consist out of some small set of dominating coefficients, while the other coefficients are small. CS theory states that an approximate reconstruction of these signals is possible, if the magnitudes of the coefficients of the signal decay fast enough, when put in descending order [7]. The error of the reconstruction depends heavily on the subrate $S$, defined as $S = M/N$ [10].
Chapter 3

Related Work

While a big part of literature on CS focuses primarily on generic signals without any additional assumption expect for sparsity, there has been significant interest in CS applied to images. The potential of CS for magnetic resonance imaging, by reducing measurement time on the patient, while maintaining a high quality, has already been well covered in the literature ([13], [14]). Also research on the use of CS for natural imagery has been made. The challenges of applying CS to natural images include the high memory and computational burden when using traditional sensing and reconstruction algorithms, due to the size of image signals and also the incorporation of image attributes, like smoothness. One of the most well-known work on CS acquisition of still images is the "single-pixel camera" [8]. The camera consists of a single photosensor and measurements of the image are made over time by projecting different binary patterns of the image pixels onto the photosensor using a digital micromirror device. Only small images up to 256x256 are considered, to avoid the huge memory and computational requirement of bigger images. For utilizing the smoothness constraint on natural images, a total variation (TV) minimization reconstruction algorithm [4] is used.
In [12] an approach to provide real-time imaging and large-scale reconstruction was made by splitting the image into blocks and applying the measurement to each block independently, which is called block-based CS (BCS). Subsequently the reconstruction from the measurements is done by the proposed algorithm, which uses Wiener Filtering and projection-based Landweber iterations to consider image smoothness and allowing a fast reconstruction. This algorithm was extended in [16] by using directional transforms in the reconstruction domain, like contourlets and complex-valued dual-tree wavelets, yielding fast reconstruction of high quality. The reconstruction quality is better than several prominent pursuits-based algorithms that do not include any smoothing and matches the quality of the popular TV minimization algorithm, while having a significantly lower computational expense.
Chapter 4

Overview of the evaluated compressed sensing measurement and reconstruction algorithms

The primary focus of the following work is to evaluate existing CS measurement and reconstruction techniques on natural images. As a measurement only BCS with a Random Gaussian Matrix is used, i.e., the blocks are measured independently with the same Random Gaussian Matrix. The tested reconstruction algorithms consist of the Basis Pursuit (BP) [2], Basis Pursuit Denoise (BPDN) [21], Matching Pursuit (MP) [15], Orthogonal Matching Pursuit (OMP) [17], Total Variation minimization with quadratic constraint (TVQC) [2]. These are applied independently to blocks of the image, as it would be too computationally exhaustive and memory demanding to reconstruct whole images with these algorithms. This is due to the size of the measurement matrix being $O(N^4)$, where $N$ is the width of the image. Additionally, the algorithm from [16], called Block Compressed Sensing with Smoothed Projected
4.1. DETAILS OF THE MEASUREMENT PROCESS

Landweber reconstruction (BCS-SPL) [16], is tested. This algorithm allows the sparsity and smoothness constraints to be adjusted across all blocks by reconstructing the image as a whole. Only the measurement process is done on each block independently for BCS-SPL.

As a transform basis a block based Discrete Cosine Transform (DCT) and block based Wavelet Transform is considered for most algorithms. For BCS-SPL complex-valued dual-tree wavelets are used on the whole image. A more detailed specification of the measurement process and a short overview of the reconstruction algorithms is now presented in this chapter.

4.1 Details of the measurement process

As noted in Chapter 2 Random Gaussian Matrices fulfill the required criteria, like coherence or RIP, that allow for a perfect or a good approximate reconstruction, with a high probability. In this work a Random Gaussian Matrix is used as the measurement matrix in a BCS manner. The grayscale image is split into blocks of size $B \times B$ and each blocks is reshaped into a vector by concatenating the rows together resulting in image block vectors $x_i$. $x$ shall now be a vector that is produced by the concatenation of all image block vectors. Subsequently each block vector is transformed (transformation is only done for MP, OMP, BP, and BPDN) and multiplied with the same Random Gaussian Matrix, resulting in a block measurement vector $y_i = \Phi_B x_i$. The Matrix $\Phi_B$ is an $M_B \times B^2$ Random Gaussian Matrix, resulting in a subrate of $S = M_B/B^2$. $\Phi_B$ is created by drawing each element from a standard normal distribution and subsequently orthogonalizing each row. A normalization of the columns was dismissed, as it showed worse reconstruction results for the Matching Pursuit algorithm while being irrelevant for the other algorithms. The columns of $\Phi_B$ still have about the same norm as the elements are picked independently from each other. The measurement matrix of the
whole image is a block diagonal matrix

$$
\Phi = \begin{bmatrix}
\Phi_B & 0 & \ldots & 0 \\
0 & \Phi_B & \ldots & 0 \\
0 & \ldots & \ddots & 0 \\
0 & \ldots & 0 & \Phi_B \\
\end{bmatrix}, \quad (4.1)
$$

As a transform matrix a block based Discrete Cosine Transform (DCT) and block based Wavelet Transform (Daubechies 2 Wavelets) is used for testing the MP, OMP, BP, and BPDN reconstruction algorithms. BCS-SPL uses complex-valued dual-tree wavelets on the whole image and the TV minimization does not need a sparsifying transform, as it assumes a sparse gradient on the image directly. Except for BCS-SPL and TV the transform matrix is also applied before the measurement matrix during the measurement step, so that the reconstruction step can be done with the measurement matrix only, i.e., \( y = \Phi \Psi x \). \( \Psi \) is also a block diagonal matrix in this case, consisting of the block transformation matrices \( \Psi_B \). BCS-SPL and TV use only the measurement matrix in the measurement step, i.e., \( y = \Phi x \) and BCS-SPL does not use a block diagonal matrix as a transform matrix.

### 4.2 Overview of the reconstruction algorithms

Except for BCS-SPL the resulting block vectors \( y_i \) are reconstructed independently from each other using the block measurement matrix \( \Phi_B \). The resulting reconstructed transformed sparse vectors \( \tilde{x}_i' \) are then transformed back into the image domain (except for TV which is already in the image domain) and concatenated together to get the final reconstructed image \( \tilde{x}' \). BCS-SPL can reconstruct the image as a whole and does not need to be executed for each block independently.
4.2. OVERVIEW OF THE RECONSTRUCTION ALGORITHMS

The different reconstruction algorithms are now shortly explained.

4.2.1 Matching Pursuit

The Matching Pursuit (MP) algorithm is an iterative greedy approach to reconstruct the sparse vector. At the beginning the residual is set to the measurement vector. In each iteration the residual is correlated with each column (atoms) of the Measurement Matrix $\Phi$ and the projection of the residual onto the atom with the highest correlation is subtracted from the residual. This is done until the residual gets small enough or a maximum number of iterations is reached [15]. In this work the algorithm stops after a parameter defined count of iterations only and uses a self-created implementation in Matlab.

4.2.2 Orthogonal Matching Pursuit

Orthogonal Matching Pursuit (OMP) is an extension for the MP algorithm. Instead of subtracting the highest correlating atom from the residual, a vector base is created by adding the highest correlating vector to this base in each iteration. The residual is created in each iteration by subtracting the projection of the measurement vector $y$ onto the vector space spanned by the created vector base from the measurement vector. In this case a different atom is picked in each iteration, as the residual will always be orthogonal to previously picked atoms, even for non orthogonal atoms [17].

For OMP it is shown, that the ideal reconstruction can be achieved if the measurement matrix fulfills boundaries for coherence or RIP and the sensed signal is sparse enough [19].

The implementation used in this work is a Matlab code from [1].
4.2. OVERVIEW OF THE RECONSTRUCTION ALGORITHMS

4.2.3 Basis Pursuit

Basis Pursuit (BP) is the classic optimization problem as already described in Chapter 2:

\[
x = \underset{\hat{x} \in \mathbb{R}^N}{\text{argmin}} \|\hat{x}\|_1, \quad s.t. \quad y = \Phi \hat{x}.
\] (4.2)

This optimization problem can be recast into a Linear Program (LP) and then solved efficiently using known LP-solvers. In this work the implementation from the \textit{l}_1-magic Matlab routine package is used (BP), which uses an interior point method for solving the optimization problem [2].

4.2.4 Basis Pursuit Denoise

In the presence of noise it is sometimes beneficial to use a relaxed version of the BP algorithm known as Basis Pursuit Denoise (BPDN), which is formulated as

\[
x = \underset{\hat{x} \in \mathbb{R}^N}{\text{argmin}} \|\hat{x}\|_1, \quad s.t. \quad \|\Phi \hat{x} - y\|_2 \leq \sigma.
\] (4.3)

The BPDN problem can be recast into a Second Order Cone Program (SOCP) and solved efficiently using known SOCP-solvers. The implementation, which is used in this work for solving the BPDN problem is a Matlab implementation from [21].

4.2.5 Total Variation minimization with quadratic constraints

The Total Variation minimization with quadratic constraints (TVQC) can be used specifically for the reconstruction of natural images, as it tries to incorporate a smoothness prior on the reconstructed signal. Let \(x_{ij}\) denote the pixel in the \(i\)th row and \(j\)th
column of an \( n \times n \) image \( X \), and define the operators,

\[
D_{h_{ij}} X = \begin{cases} 
X_{i+1,j} - X_{ij} & i < n \\
0 & i = n 
\end{cases}, (4.4)
\]

and

\[
D_{v_{ij}} X = \begin{cases} 
X_{i,j+1} - X_{ij} & j < n \\
0 & j = n 
\end{cases}. (4.5)
\]

Then the Total Variation of the image \( X \) is

\[
TV(X) := \sum_{ij} \sqrt{(D_{h_{ij}} X)^2 + (D_{v_{ij}} X)^2}. (4.6)
\]

So the TV is simply the sum of the magnitudes of the discrete gradient at every point of the image. The reconstruction problem is then specified as

\[
\hat{x} = \arg\min_{\hat{x} \in \mathbb{R}^N} TV(X(\hat{x})), \quad s.t. \quad \|\Phi \hat{x} - y\|_2 \leq \sigma. (4.7)
\]

Again this problem can be recast to a SOCP. To solve this problem a Matlab implementation from [2] is used, which utilizes log-barrier iterations.

### 4.2.6 Block Compressed Sensing with Smoothed Projected Landweber reconstruction

Another algorithm specifically designed for the reconstruction of natural images is the Block Compressed Sensing with Smoothed Projected Landweber reconstruction (BCS-SPL) from [16]. It tries to solve the problem of the long execution times of the TV minimization, while still incorporating image priors like smoothness. As a transform matrix complex-valued dual-tree wavelets are used on the whole image, to exploit
sparsity across the whole image and prevent blocking artifacts. The implementation used in this work is from [9].
Chapter 5

Parameter exploration of the evaluated compressed sensing measurement and reconstruction algorithms

Some of the reconstruction algorithms presented in the previous chapter can be customized by parameters. In this chapter an exploration of the parameters is done by measuring the reconstruction quality of the algorithms with different parameter settings. At first the iterations count parameter for MP and OMP is examined. In the second part of this chapter the effect of the threshold parameter $\sigma$ in BPDN and TVQC is investigated. To estimate the behaviour of the parameters, 15 natural images, taken from the KODAK image database [11], are sensed with a fixed Random Gaussian Matrix with different subrates and a block based DCT as a transformation matrix.
5.1 Effects of iteration count in Matching Pursuit and Orthogonal Matching Pursuit

Matching Pursuit and Orthogonal Matching Pursuit both have an iteration count parameter, specifying the amount of matching steps that are executed. The sensed images are reconstructed with the MP and OMP algorithms with different iteration parameters \( I \), which specify the amount of matching steps relative to the blocksize. In the end the Peak-Signal-To-Noise-Ratio \( PSNR \) between the reconstructed images and the ground truth images is calculated. The results can be seen in fig. 5.1. The results are relatively equal for both block sizes, with the \( 32 \times 32 \) blocksize showing a little bit better PSNR values overall. The MP algorithms improves monotonically with a higher iteration count, converging to a maximum value, while the OMP reconstruction degrades if the iteration parameter is chosen too small or too large, except for really high subrates. The maxima of the OMP algorithm are nearly equal to the maxima of the MP algorithm for lower subrates and exceed MP for higher subrates, starting from \( S = 0.7 \). The ideal iteration parameter of the OMP algorithm depends on the subrate, as it increases for an increasing subrate. For the following experiments the ideal iteration parameter for OMP is therefore approximated by \( I_{\text{omp}} = S^3 \), which shows to be a good regression for the ideal parameter. For MP the relative iteration count is set to \( I_{\text{mp}} = 0.8 \) in the following, as the convergence seems to be sufficient at this point.
5.1. EFFECTS OF ITERATION COUNT IN MATCHING PURSUIT AND ORTHOGONAL MATCHING PURSUIT

**Figure 5.1:** Plots between I and PSNR for different subrates $S$ of MP and OMP. Block sizes are $16 \times 16$ and $32 \times 32$ and the transform matrix is the DCT.
5.2 Effects of threshold parameter $\sigma$ in Basis Pursuit Denoise and Total Variation with Quadratic Constraint

Basis Pursuit Denoise (BPDN) and Total Variation with Quadratic Constraint (TVQC) both require a threshold parameter $\sigma$, specifying the allowed deviation from the given measurement vector to the measurement vector, that would result if the reconstructed image would be sensed again. As in section 5.1 different combinations of threshold parameters and subrates are tested for both algorithms and the corresponding PSNR is calculated. The results are shown in fig. 5.2 for BPDN and in fig. 5.3 for TVQC. For BPDN the deviation of the PSNR for different sigmas between 0 and 0.2 is really low. For a subrate of 0.4 the maximum gets reached at $\sigma = 0$ and a subrate of 0.7 the maximum gets reached at $\sigma = 0.001$. The threshold parameter $\sigma$ for the following experiments is therefore chosen to be 0.001, giving nearly exactly the same results as BP, which uses the equality constraint. TVQC seems to be degrading with a bigger $\sigma$, regardless of the subrate and therefore the threshold parameter $\sigma$ is also chosen to be 0.001 for TVQC in the following.
5.2. Effects of threshold parameter $\sigma$ in Basis Pursuit Denoise and Total Variation with Quadratic Constraint

**Figure 5.2**: Plots between $\sigma$ and PSNR of BPDN for different subrates $S$. Blocksize is $32 \times 32$ and the transform matrix is the DCT.
5.2. EFFECTS OF THRESHOLD PARAMETER $\sigma$ IN BASIS PURSUIT
DENOISE AND TOTAL VARIATION WITH QUADRATIC CONSTRAINT

Figure 5.3: Plots between $\sigma$ and PSNR of BPDN for different subrates $S$. Blocksize is $32 \times 32$ and the transform matrix is the DCT.
Chapter 6

Comparison of the different compressed sensing measurement and reconstruction algorithms

With the optimized parameters from Chapter 5, the algorithms are now tested against each other. The measurement process is done like specified in Section 4.1.

Figure 6.1 shows the plot of the PSNR quality metric against the subrate of the measurement process for each algorithm. It can be seen, that the DCT transform matrix is better suited than the Daubechies 2 Wavelets, as all algorithms, that utilize the transform matrix during the measurement, show a better quality with DCT. The quality of MP, BP and BPDN are quite similar. The OMP reconstruction performs worse than all other algorithms. This could be mitigated by choosing the optimal iteration parameter $I$, which is only approximated in this experiment. The best algorithms are the TVQC and BCS-SPL outperforming the other reconstructions significantly. TVQC
and BCS-SPL perform similarly. TV seems to have a small advantage for high subrates and the $32 \times 32$ blocksize.

Figure 6.1: PSNR of the different reconstruction algorithms for specific subrates.
In fig. 6.2 one of the reconstructed images [11] (kodim07.png) for a subrate of 0.4 can be seen for all algorithms. As a transform matrix the DCT is used, except for TV and BCS-SPL. Artifacts appear especially around the detailed flower in the foreground. Blocking artifacts are clearly visible for all reconstructions except for TV and BCS-SPL.

Figure 6.2: Reconstructed images for a subrate $S = 0.4$ and a blocksize $B = 32 \times 32$ and the original groundtruth image.

In the end execution time measurements have been done for each algorithm on a workstation with an Intel(R) Core(TM) i5-4690 CPU @ 3.50GHz and 16 GB RAM. It has to be noted, that the execution time also depends on the implementation, although
are implemented in Matlab and single threaded, and only some of the algorithms are optimized for a low execution time. The results can be seen in fig. 6.3. Even though BCS-SPL delivers the best reconstruction quality, it still has the lowest execution time, followed by OMP and MP. As previously mentioned, the TVQC algorithms is quite computationally exhaustive.

<table>
<thead>
<tr>
<th>Execution times in s</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>MP</td>
<td>11.8</td>
</tr>
<tr>
<td>OMP</td>
<td>8.3</td>
</tr>
<tr>
<td>BP_l1magic</td>
<td>42.4</td>
</tr>
<tr>
<td>BPDN</td>
<td>79.0</td>
</tr>
<tr>
<td>TVQC</td>
<td>287.6</td>
</tr>
<tr>
<td>BCS-SPL</td>
<td>4.8</td>
</tr>
</tbody>
</table>

**Figure 6.3:** Execution times for the reconstruction of image kodim07.png with subrate $S = 0.4$ and blocksize $B = 32 \times 32$. 
Chapter 7

Conclusions and Outlook

The main goal of this work was to get familiar with the compressed sensing framework and evaluate its suitability on the measurement and reconstruction of natural images. One problem, when dealing with images is the high dimensionality of the signal, leading to a demanding computational effort and memory requirement. To deal with this a block based approach was inspected. Here a strict independent reconstruction of each block, which was used for the MP, OMP, BP, BPDN and TVQC reconstruction, or an independent sensing of each block with a common reconstruction, which was used in the BCS-SPL reconstruction, is possible. Without incorporating image priors, by just using the basic reconstruction algorithms like MP, OMP, BP or BPDN the reconstruction quality is relatively low. Reconstruction algorithms like TVQC or BCS-SPL, which focus on the reconstruction of images show improved results. The block independent sensing results in blocking artifacts for all algorithms, but especially for MP, OMP, BP and BPDN. BCS-SPL shows a low execution time compared to the other algorithms, especially in contrast to TVQC, BP and BPDN.

In the future work, the principles of TV and BCS-SPL reconstruction could be
inspected further. Existing algorithms, dealing with similar measurement and reconstruction problems on images, like the Frequency Selective Reconstruction (FSR) [18], which were originally developed outside of the compressed sensing framework, could potentially profit from these methods or vice versa. The FSR focuses on the reconstruction of an image from a non-regular sampled grid to a regular grid. The measurement matrix in this case is different from the block based Random Gaussian Matrix used in this work. Further evaluations on the effect of this measurement matrix, corresponding to the unregular sampling, have to be evaluated.
Bibliography


